ELECTROHYDRODYNAMIC FLOW BETWEEN PARALLEL DIELECTRIC CHANNELS

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Steady, accelerated, and pulsating electrodynamic flows in a plane dielectric channel are considered, along with Couette flow. It is shown that for these types of electrohydrodynamic flows the effect is concentrated in a thin layer near the walls, which can considerably change the friction stress on the walls. Some exact solutions of the energy equation are obtained.

I. Flow of an incompressible fluid with a unipolar charge between two parallel dielectric plates in a longitudinal electric field E_{0x} is examined. In a single-fluid approximation, the corresponding system of equations of electrohydrodynamics (EHD) for small Reynolds numbers $R_e = U/bE_{0x} \ll 1$ has the form [1]

$$\frac{\partial P}{\partial x} = \mu \frac{d^2u}{dy^2} + \varepsilon E_{0x} \frac{dE_y}{dy}; \tag{1}$$

$$\frac{\partial P}{\partial y} = \varepsilon E_y \frac{dE_y}{dy}; \tag{2}$$

$$bE_y \frac{dE_y}{dy} - D \frac{d^2E_y}{dy^2} = 0. \tag{3}$$

The pressure P in this case can be represented as

$$P = \frac{\partial P}{\partial x} x + \frac{\varepsilon}{2} E_y^2,$$

if we bear in mind that u and E_y are independent of x.

Assuming that the solid dielectric does not have surface or volume conductivity, we write the boundary conditions for the electric field as follows (the coordinate origin is on the channel axis):

$$E_y|_{y=0} = 0; \quad E_y|_{y=h} = \frac{I}{\varepsilon b E_{0x}}. \tag{4}$$

We introduce the dimensionless quantities

$$\bar{E}_y = \frac{\varepsilon b E_y E_{0x}}{I}; \quad \bar{y} = \frac{y}{h}; \quad \mathrm{Pe_e} = \frac{Ih}{\varepsilon D E_{0x}};$$

$$\bar{q} = \frac{qbhE_{0x}}{I}; \quad \frac{h^2}{\mu U}\frac{\partial P}{\partial x} = A; \quad \mathrm{Eu_e} = \frac{Ih}{b\mu U},$$

where

$$I = \int_0^h bE_{0x} q\, dy = \varepsilon b E_{0x} E_y|_{y=h}; \quad q = \varepsilon \frac{dE_y}{dy}; \quad U = u|_{y=0}.$$

Taking (4) into account and integrating Eq. (3), we find

$$\bar{E}_y = \sqrt{C}\,\mathrm{tg}\, C_1\bar{y}; \quad \bar{q} = \frac{\mathrm{Pe_e} C}{2\cos^2 C_1\bar{y}}, \tag{5}$$

where the constant of integration C is determined from

$$1 = \sqrt{C}\,\mathrm{tg}\, C_1, \quad C_1 = \frac{\mathrm{Pe_e}\sqrt{C}}{2}. \tag{6}$$

1. **Steady EHD flow in a plane channel.** Consider the case in which the hydrodynamic velocity and electric body force qE_{0x} have opposite directions. This case is interesting in that flows similar to detached flows can arise in the channel when $du/dy|_{y=h} = 0$. The problem of similar flow when the directions of u and E_{0x} coincide was examined in [1].

If we write Eq. (1) for this case in dimensionless form,

$$\frac{d^2\bar{u}}{d\bar{y}^2} = A - \frac{\mathrm{Eu_e Pe_e} C}{2\cos^2 C_1\bar{y}}$$

and integrate it under the usual boundary conditions, we obtain

$$\bar{u} = -\frac{A}{2}(1-\bar{y}^2) + \frac{2\mathrm{Eu_e}}{\mathrm{Pe_e}} \ln \frac{\cos C_1\bar{y}}{\cos C_1}, \tag{7}$$

from which it follows that $d\bar{u}/d\bar{y}|_{\bar{y}=0;1} = 0$ implies $A = -\mathrm{Eu_e}$. When $\mathrm{Eu_e} > -A$, the velocities near the walls and at the center of the channel have opposite directions. The extreme points on the velocity profile in this case are given by

$$A\bar{y}_e + \mathrm{Eu_e}\sqrt{C}\,\mathrm{tg}\, C_1\bar{y}_e = 0.$$

"Squeezing" of the velocity profile near the walls is due to the strong EHD effect produced by the concentration of high space-charge densities in these regions. The degree of nonuniformity of the space-charge density distribution can be estimated from (5), assuming, for example, $\mathrm{Pe_e} \approx 10^3$, which corresponds to real EHD flows:

$$\frac{\bar{q}|_{\bar{y}=1}}{\bar{q}|_{\bar{y}=0}} \sim 10^5.$$

The flow rate of the fluid through the channel cross section in the absence of the EHD effect is determined in the well-known way:

$$\overline{M}=\int_{-1}^{1}\overline{u}d\overline{y}=-\frac{2}{3}A.$$

Here

$$\overline{M}=\frac{M}{hU};\quad M=\int_{-h}^{h}udy.$$

If the velocity profile is represented by expression (7), then

$$\overline{M}=-\frac{2}{3}A-\frac{2\mathrm{Eu_e}}{\mathrm{Pe_e}}\left[2\ln\cos C_1-\sum_{n=1}^{\infty}\frac{(2^{2n}-1)B_n\mathrm{Pe}_e^{2n}C^n}{n!\,(2n+1)!}\right].\tag{8}$$

The second term in (8) determines the reduction in flow rate (B_n are Bernoulli numbers).

For the electrohydrodynamic analog of Couette flow, the space-charge distribution coincides with the space-charge distribution in a channel with fixed walls, and the equation of motion in the presence of an external field $\pm E_{0x}$ has the form

$$\frac{d^2\overline{u}}{d\overline{y}^2}=\mp\frac{\mathrm{Eu_e\,Pe_e}C}{2\cos^2C_1(\overline{y}-1)}.\tag{9}$$

The boundary conditions are

$$\overline{u}=-\frac{U_1}{U_0}\quad\text{at}\quad\overline{y}=0;$$

$$\overline{u}=1\quad\text{at}\quad\overline{y}=2.$$

The coordinate origin is on the lower plate, which moves with velocity $-U_1$.

The solution of Eq. (9) has the form

$$\overline{u}=\frac{\overline{y}}{2}\left(1+\frac{U_1}{U_0}\right)-\frac{U_1}{U_0}\pm\frac{2\mathrm{Eu_e}}{\mathrm{Pe_e}}\ln\frac{\cos C_1(\overline{y}-1)}{\cos C_1}.\tag{10}$$

The point $\overline{y}_0$ at which the flow velocity is zero is determined from the following transcendental equation:

$$\frac{\overline{y}_0}{2}\left(1+\frac{U_1}{U_0}\right)-\frac{U_1}{U_0}=\mp\frac{2\mathrm{Eu_e}}{\mathrm{Pe_e}}\ln\frac{\cos C_1(\overline{y}_0-1)}{\cos C_1}.$$

For the friction stress $\overline{\tau}=d\overline{u}/d\overline{y}$ (here $\overline{\tau}=\tau h/\mu U_0$) we obtain the expression

$$\overline{\tau}=\frac{1}{2}\left(1+\frac{U_1}{U_0}\right)\pm\mathrm{Eu_e}\sqrt{C}\,\mathrm{tg}\,C_1(\overline{y}-1),\tag{11}$$

from which it follows that when the direction of force qE_{0x} is positive the decrease in friction on the upper wall and the increase on the lower wall are determined by the second term in (11). If the absolute value of this term is greater than $(1 + U_1/U_0)/2$, there will be an extreme point on the velocity profile near one of the walls.

It should be noted that in [1], from a consideration of the EHD analog of Poiseuille flow, it was concluded that the EHD effect on the maximum velocity in the channel was slight, which is a consequence of the above-mentioned great nonuniformity of the space-charge distribution.

The author extrapolates his estimates to the boundary layer and expresses a doubt that the EHD effect on boundary-layer separation is substantial.

Such extrapolation is of questionable validity since the pressure gradients in a boundary layer near the separation point and in a plane channel have opposite signs, and the space-charge distribution in a boundary layer, which is to a considerable extent dependent on the external electric field and the physical properties of the surface, can differ considerably from the charge distribution in an infinite channel [2]. In addition, the determining factor is not the maximum value of the ponderomotive component of the velocity on the channel axis but the variation of the velocity gradient at the wall, which, in the final analysis, determines separation. The above examples indicate that this value varies fairly substantially.

2. Accelerated channel flow. Such flow is produced when the EHD effect is suddenly applied to a steady flow. We assume that expressions (5) remain valid. The corresponding equation of motion in the presence of $\pm E_{0x}$ has the form

$$\frac{\partial\overline{u}}{\partial\overline{t}}+\frac{\partial^2\overline{u}}{\partial\overline{y}^2}=-A\pm\frac{\mathrm{Eu_e\,Pe_e}C}{2\cos^2C_1(\overline{y}-1)}.\tag{12}$$

The boundary conditions (the coordinate origin is on the lower surface of the channel) are

$$\overline{u}|_{\overline{t}=0}=-A\overline{y}\left(1-\frac{\overline{y}}{2}\right);$$

$$\overline{u}|_{\overline{y}=0}=0;\quad\overline{u}|_{\overline{y}=2}=0;\quad\left(\overline{t}=\frac{t\nu}{h^2}\right).\tag{13}$$

The solution of Eq. (12) with account for (13) has the form

$$\overline{u}=-A\overline{y}\left(1-\frac{\overline{y}}{2}\right)\pm\frac{2\mathrm{Eu_e}}{\mathrm{Pe_e}}\ln\frac{\cos C_1(\overline{y}-1)}{\cos C_1}\pm$$
$$\pm\frac{8\mathrm{Eu_e}}{\pi\,\mathrm{Pe_e}}\ln\cos C_1\sum_{n=0}^{\infty}\frac{1}{2n+1}\exp\left[-\frac{\pi^2}{4}(2n+1)^2\overline{t}^2\right]\times$$
$$\times\sin\frac{\pi}{2}(2n+1)\overline{y}\pm$$
$$\pm\frac{4\mathrm{Eu_e}}{\mathrm{Pe_e}}\sum_{n=0}^{\infty}\sum_{k=1}^{\infty}\sum_{i=1}^{\infty}\Big\{(-1)^{k+1}B_{2k}\mathrm{Pe}_e^{2k}C^k\sin(2n+1)\pi\overline{y}\times$$
$$\times\exp\left[-\frac{\pi^2}{4}(2n+1)^2\overline{t}\right]\times$$
$$\times\{k\pi^{2i+1}2^{2k-2i-1}(2k-2i-1)(2n+1)^{2i+1}\}^{-1}\Big\}.\tag{14}$$

As might be expected, when $\overline{t}\to\infty$, the flow approaches the steady state, which is described by the equation

$$\overline{u}|_{\overline{t}\to\infty}=-A\overline{y}(1-\overline{y})\pm\frac{2\mathrm{Eu_e}}{\mathrm{Pe_e}}\ln\frac{\cos C_1(\overline{y}-1)}{\cos C_1}.$$

3. Steady, pulsating channel flow. Such flow can be caused by a variable electric field $E_{0x} = E_0\cos\omega t$. We restrict ourselves to the case in which the space-charge distribution over the channel cross section is not a function of time, which is possible when

$$\int_0^h qdy=\frac{I(t)}{bE_{0x}(t)}=Q_1.\tag{15}$$

Here, the solution of the electric-current continuity equation (3) has the form of (5) and can be written as follows:

$$q = \frac{Q_1 \mathrm{Pe_e} C}{2h \cos^2 C_1 y},$$

where

$$\mathrm{Pe_e} = \frac{I_0 h}{\varepsilon D E_0}, \quad I_0 = \max |I|.$$

The initial equation of motion has the form

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{Q_1 \mathrm{Pe_e} E_0 \cos \omega t}{2\rho h \cos^2 C_1 y}. \qquad (16)$$

The initial condition has no meaning in the given problem; the boundary conditions are conventional.

Having written $E_{0x} = E_0 e^{i\omega t}$, we seek the solution in the form

$$u = f(y) e^{i\omega t}. \qquad (17)$$

Substituting (17) into (16), we arrive at the equation

$$f'' - \frac{i\omega}{\nu} f + \frac{Q_1 E_0 \mathrm{Pe_e} C}{2\rho h \cos^2 C_1 y} = 0 \qquad (18)$$

with the boundary conditions

$$f = 0 \quad \text{at} \quad y = \pm h.$$

Solving (18) by the variation-of-constants method and taking (17) into account, we obtain

$$u = \frac{Q_1 C \mathrm{Pe_e} E_0 \exp i\omega t}{2\mu h \sqrt{i\omega\nu}} \left[\int_0^y \frac{\mathrm{sh}(y-\xi)\sqrt{\frac{i\omega}{\nu}}}{\cos^2 \frac{C_1 \xi}{h}} d\xi - \frac{\mathrm{ch}\, y \sqrt{\frac{i\omega}{\nu}}}{\mathrm{ch}\, h \sqrt{\frac{i\omega}{\nu}}} \int_0^h \frac{\mathrm{sh}(h-\xi)\sqrt{\frac{i\omega}{\nu}}}{\cos^2 \frac{C_1 \xi}{h}} d\xi \right] \qquad (19)$$

Only the real part has physical meaning in this expression. If we use the expansions of sh and ch into series, calculate integrals (19), and isolate the real part, we obtain the final solution of the problem, which, because of its cumbersomeness, is not given here.

II. Let us consider some exact solutions of the energy equation for EHD flow between parallel plates when the physical constants of the fluid are independent of temperature and the initial hydrodynamic and thermal intervals are ignored. The corresponding equation is written in dimensionless form as follows [2]:

$$\frac{1}{\mathrm{Fo}} \frac{\partial \bar{T}}{\partial \bar{t}} + \mathrm{Pe} \frac{\partial \bar{T}}{\partial \bar{x}} = \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \mathrm{S} \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 + \mathrm{S_e} \bar{q}. \qquad (20)$$

1. Steady EHD flow between two parallel plates heated to temperature T_0. The heat-flux equation in this case has the form

$$\frac{d^2 \bar{T}}{\partial \bar{y}^2} + \mathrm{S} \left(\frac{d\bar{u}}{\partial \bar{y}} \right)^2 + \mathrm{S_e} \bar{q} = 0. \qquad (21)$$

The boundary conditions are

$$\bar{T} = 1 \quad \text{at} \quad \bar{y} = 0;\ 2. \qquad (22)$$

If the lower plate is fixed and the upper plate moves with velocity U_0, the temperature distribution over the width of the channel is, on the basis of (21), (22) and (10), described by the following expression:

$$T = 1 + 0{,}125\mathrm{S}\, \bar{y}(2-\bar{y})(1 + 4\mathrm{Eu_e^2} C) \pm \pm \frac{2}{\mathrm{Pe_e}} \left(\frac{2\mathrm{Eu_e^2 S}}{\mathrm{Pe_e}} + \mathrm{S_e} \right) \ln \frac{\cos C_1 (\bar{y}-1)}{\cos C_1} \pm \pm \mathrm{Eu_e S} \sum_{n=1}^{\infty} \frac{(2^{2n}-1) B_n \mathrm{Pe_e^{2n-1}} C^n}{n \cdot (2n+1)!} \left[1 + (\bar{y}-1)^{2n+1} - \bar{y}\right]. \qquad (23)$$

When $\mathrm{Eu_e} = \mathrm{S_e} = 0$, expression (23) coincides with the corresponding solution for conventional hydrodynamics.

If the charged fluid moves between fixed parallel dielectric plates, then, taking (22) and the corresponding expression for the velocity profile into account, the solution of Eq. (21) has the form

$$\bar{T} = 1 + \frac{\mathrm{S}}{b} \bar{y}(2-\bar{y})[A^2(2+\bar{y}) + 3\mathrm{Eu_e^2} C] \pm \pm 2A\, \mathrm{Eu_e S} \sum_{n=1}^{\infty} \frac{(2^{2n}-1)}{n(2n+1)} \mathrm{Pe_e^{2n-1}} C^n [1 + (\bar{y}-1)^{2n+1} - \bar{y}] + + \frac{2}{\mathrm{Pe_e}} \left(\frac{2\mathrm{Eu_e S}}{\mathrm{Pe_e}} + \mathrm{S_e} \right) \ln \frac{\cos C_1 (\bar{y}-1)}{\cos C_1}. \qquad (24)$$

Let us estimate the relationship between the Joule heat and the heat due to friction. On the basis of (23) and (24), assuming that $\mathrm{Pe_e} \approx 10^3$ and $\mathrm{tg}(\mathrm{Pe_e}\sqrt{C}/2) \approx \approx 2/(\pi - \mathrm{Pe_e}\sqrt{C})$, we can write, respectively,

$$\bar{T}|_{\bar{y}=1} \approx 1 + 0{,}125\mathrm{S} \left[1 + \frac{2\pi\, \mathrm{Eu_e^2}}{(2+\mathrm{Pe_e})^2} \right] \mp \mp \frac{2}{\mathrm{Pe_e}} \left(\frac{2\mathrm{Eu_e^2 S}}{\mathrm{Pe_e}} + \mathrm{S_e} \right) \ln \frac{\pi}{2+\mathrm{Pe_e}};$$

$$\bar{T}|_{\bar{y}=1} \approx 1 + \frac{\mathrm{S}}{2} \left[A^2 + \frac{2\pi\, \mathrm{Eu_e^2}}{(2+\mathrm{Pe_e})^2} \right] \mp \mp \frac{2}{\mathrm{Pe_e}} \left(\frac{2\mathrm{Eu_e^2 S}}{\mathrm{Pe_e}} + \mathrm{S_e} \right) \ln \frac{\pi}{2+\mathrm{Pe_e}}.$$

It is apparent from these expressions that when $\mathrm{Eu_e} \approx \approx A \approx 1$, $\mathrm{S} \approx 10^{-3}$, and $\mathrm{S_e} \approx 10^{-2}$, heating of the fluid on the channel axis due to Joule heat is much less than the heating due to friction. This does not hold true near the walls, where the space charge is concentrated and Joule losses are great.

2. Lower channel wall at rest while upper moves in positive direction with velocity $\bar{u} = 1$. Up to the initial time, both walls are at temperature $\bar{T} = 0$. At time $\bar{t} = 0$, the upper wall is instantaneously heated to temperature $\bar{T} = 1$. We must find the temperature distribution at $\bar{t} > 0$.

The heat flux equation in this case has the form

$$\frac{1}{\mathrm{Fo}} \frac{\partial \bar{T}}{\partial \bar{t}} = \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \mathrm{S} \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 + \mathrm{S_e} \bar{q}. \qquad (25)$$

The boundary conditions are

$$\overline{T}(\overline{y},\ 0)=0;\quad \overline{T}(0,\ \overline{t})=0;\quad \overline{T}(2,\ \overline{t})=1. \tag{26}$$

Substituting the corresponding expressions for $\overline{u}$ and $\overline{q}$ into (25), we obtain

$$\frac{1}{\mathrm{Fo}}\frac{\partial\overline{T}}{\partial\overline{t}}=\frac{\partial^2\overline{T}}{\partial\overline{y}^2}+\mathrm{S}\ \left[0{,}5+\mathrm{Eu_e}\sqrt{\overline{C}}\operatorname{tg}C_1(\overline{y}-1)^2\right]+ +\frac{C\,\mathrm{S_ePe_e}}{2\cos^2 C_1(\overline{y}-1)}, \tag{27}$$

whose solution is sought as

$$\overline{T}=\overline{T}_0+\overline{T}_1, \tag{28}$$

where $\overline{T}_0$, in turn, satisfies the equation

$$\frac{d^2\overline{T}_0}{d\overline{y}^2}=\mathrm{S}\left[0{,}5-\mathrm{Eu_e}\sqrt{C}\operatorname{tg}C_1(\overline{y}-1)^2\right]+\frac{C\,\mathrm{S_ePe_e}}{2\cos^2 C_1(\overline{y}-1)}$$

and boundary conditions (26).

The expression for $\overline{T}_0$ has the form

$$\overline{T}_0=0{,}5\overline{y}+0{,}125\mathrm{S}\,\overline{y}(2-\overline{y})(1+4\mathrm{Eu_e^2}C)\pm \pm\mathrm{Eu_eS}\sum_{n=1}^{\infty}\frac{(2^{2n}-1)B_n}{n\cdot(2n+1)!}\mathrm{Pe_e^{2n-1}}C^n\left[1+(\overline{y}-1)^{2n}-\overline{y}\right]+ +\frac{2}{\mathrm{Pe_e}}\left(\frac{2\mathrm{Eu_e^2S}}{\mathrm{Pe_e}}+\mathrm{S_e}\right)\ln\frac{\cos C_1(\overline{y}-1)}{\cos C_1}. \tag{29}$$

Substituting (28) and (29) into (26) and (27), we arrive at the first boundary-value problem for the equation of thermal conductivity:

$$\frac{\partial\overline{T}_1}{\partial\overline{t}}=\mathrm{Fo}\frac{\partial^2\overline{T}_1}{\partial\overline{y}^2}, \tag{30}$$

$$\overline{T}_1(0,\ \overline{t})=0,$$

$$\overline{T}_1(2,\ \overline{t})=1,$$

$$\overline{T}(\overline{y},\ 0)=-\overline{T}_0.$$

Solving (30) with allowance for (28), after transformations we obtain

$$\overline{T}=0{,}5\overline{y}+0{,}125\mathrm{S}\,\overline{y}(2-\overline{y})(1+4\mathrm{Eu_e^2}C)\pm \pm\mathrm{Eu_eS}\sum_{n=1}^{\infty}\frac{(2^{2n}-1)B_n}{n\cdot(2n+1)!}\mathrm{Pe_e^{2n-1}}C^n\left[1+(y-1)^{2n+1}-\overline{y}\right]+ +\frac{2}{\mathrm{Pe_e}}\left(\frac{2\mathrm{Eu_e^2S}}{\mathrm{Pe_e}}+\mathrm{S_e}\right)\ln\frac{\cos C_1(\overline{y}-1)}{\cos C_1}+ +2\sum_{k=0}^{\infty}\sin\frac{\pi(k+1)}{2}\overline{y}\exp\left(-\mathrm{Fo}^2\frac{\pi^2k^2\overline{t}^2}{4}\right)\times \left\{\frac{(-1)^k}{\pi(k+1)}+\frac{(1+4\mathrm{Eu_e^2}C)}{\pi^3(2k+1)^3}\pm\right. \pm\mathrm{Eu_eS}\sum_{n=1}^{\infty}\frac{(2^{2n}-1)B_n}{n\cdot(2n+1)!}\mathrm{Pe_e^{2n-1}}C^n \left[\frac{1}{\pi(2n+1)}-\frac{2}{\pi n}(-1)^{n+1}\pm\right. \left.\left.\pm\sum_{i=0}^{2n}\sum_{m=0}^{i}(-1)^{3n+m-1}\binom{2n}{i}\binom{i}{2m}\cdot(2m)!\,\frac{2i}{(\pi n)^{2n+1}}\right]\right\}. \tag{31}$$

Here, $\binom{a}{b}$ are binomial coefficients.

On the basis of expression (31), it can be shown that the third term in the braces has a value of $\sim\mathrm{Eu_eS}/\mathrm{Pe_e^2}$ and when $\overline{t}\ll 1$, $\overline{y}=1$, $\mathrm{Eu_e}\approx 1$, and $\mathrm{Pe_e}\approx 10^3$, just as in the steady case, the Joule heat is much less than the heat due to friction. Joule losses are greatest near the channel walls.

CONCLUSIONS

Flow of incompressible fluid with a unipolar charge between two parallel dielectric plates in a longitudinal electric field is considered. Problems of electrohydrodynamic Couette flow (10), steady, accelerated, and pulsating flows in a plane channel (7), (14), and (19), respectively, are solved. Exact solutions (23), (24), and (31) are obtained for the energy equation for incompressible Couette flow between moving and fixed walls.

NOTATION

u(y) is the flow velocity, P is the pressure, μ and ν are the dynamic and kinematic viscosity coefficients, E_{0x} and E_y are the strengths of external electric field and that produced by space charge, b is the electric-charge mobility, D is the charge diffusion coefficient, h is the channel half-width, U_0 and U_1 are the motion velocities of upper and lower plates, t is the time, T is the temperature (°K), $\overline{T}=(T-T_l)/(T_u-T_l)$, T_l and T_u are the temperatures of lower and upper plates, $S=\mu U_0^2/\rho c_p(T_u-T_l)$ is Schlichting's number, $S_e=Ih/\varkappa(T_u-T_l)$ is the electric analog of Schlichting's number, a is the thermal diffusivity, $\varkappa$ is the thermal conductivity, c_p is the specific heat, ρ is the fluid density.

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